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Nothing anomalous about two-loop HBCPT: consistency with the LET for spin-dependent Compton scattering

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Abstract

The leading order contributions of processes involving anomalous pion-photon vertices to forward spin-dependent Compton scattering from nucleons are considered in heavy baryon chiral perturbation theory. These all involve the exchange of three pions between one or two photons and the nucleon, and hence are two-loop processes. We find that the sum of these processes vanishes in the manner predicted by the low energy theorem of Low, Gell-Mann and Goldberger as the photon energy goes to zero. This provides the first consistency test of two-loop HBCPT.

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Compton scattering from the nucleon has recently been the subject of much work, both experimental and theoretical. For the case of unpolarised protons the experimental amplitude is well determined, and in good agreement with the results of heavy baryon chiral perturbation theory (HBCPT). However the situation with regard to scattering from polarised targets is less satisfactory. The usual notation for spin-dependent pieces of the forward scattering amplitude for photons of energy ω, momentum q is

$$\epsilon_2^\mu \Theta_{\mu\nu} \epsilon_1^\nu = i e^2 \omega W^{(1)}(\omega) \boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_1) + \ldots \tag{1}$$

From a theoretical perspective there is particular interest in the low energy limit of the amplitude: $W^{(1)}(\omega) = 4\pi(f_2(0) + \omega^2 \gamma_0) + \ldots$, where γ_0 is the forward spin-polarisability. The low energy theorem (LET) of Low, Gell-Mann and Goldberger [1] states that $f_2(0) = -e^2\kappa^2/8\pi M_N^2$.

Until very recently no direct measurements of polarised Compton scattering had been attempted, but preliminary data now exist from MAMI at Mainz, for photon energies between 200 and 800 MeV; the range will be extended downward to 140 MeV, and a future experiment at Bonn will extend it upwards to 3 GeV [2]. In order to make contact with the low energy value—which will not be measured directly in the near future—use is made of two sum rules: the Gerasimov-Drell-Hearn (GDH) sum rule [3]

$$f_2(0) = \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_-(\omega) - \sigma_+(\omega)}{\omega} d\omega, \tag{2}$$

where $\sigma_\pm$ are the parallel and antiparallel cross-sections, and ω_0 is the threshold for pion production; and the related sum rule for γ_0 which has the same form except that $1/\omega^3$ replaces $1/\omega$.

Before direct data existed, the relevant cross-sections were estimated from multipole analyses of pion electroproduction experiments [4,5]. These showed significant discrepancies between the LET and the GDH sum rule for the difference of $f_2(0)$ for the proton and neutron, though the sum was in good agreement. Indeed even the sign of the difference was different. The preliminary data from MAMI [2] suggest a continuing discrepancy between the LET and the sum rule for the proton, though a smaller one than given by the multipole analysis.

There is also a prediction from HBCPT for γ_0: at lowest (third) order in the chiral expansion $\gamma_0 = e^2 g_A^2/(96\pi^3 f_\pi^2 m_\pi^2) = 4.4 \times 10^{-4}$ fm^4 for both proton and neutron. Higher order contributions have not yet been calculated, though the effect of the Δ, which enters in counter-terms at fifth order, has been estimated to be so large as to change the sign [6]. The calculation has also been done in a generalised HBCPT with an explicit Δ [7]; the effect is smaller, giving 2.2×10^{-4} fm^4 in total. Multipole analysis of electroproduction data actually gave a negative polarisability, in strong contradiction to the lowest order chiral result; the MAMI data does not currently go low enough in energy to give a reliable result. Clearly we have in γ_0 a quantity for which a two-loop analysis will be crucial [5].

Chiral perturbation theory [8] is establishing itself as the principal tool for determining the consistency of data in disparate low energy processes involving pions, nucleons and photons. The purely mesonic theory is now on a very firm footing, and two-loop calculations are becoming commonplace [9]. Originally it appeared impossible to include nucleons consistently, as the existence of the nucleon mass as an extra mass scale destroys the power

counting of the relativistic theory [10]. This problem can be circumvented, however, by expanding about the limit in which the nucleon mass is infinitely large, generating a systematic expansion in which M_N occurs only in the denominator. This theory has been extensively tested at one-loop order, proving consistent with—and indeed providing another method of demonstrating—all low energy theorems based on such considerations as Lorentz and gauge invariance and chiral symmetry.

However until now calculations in HBCPT have been almost exclusively one-loop, for the excellent reason that two-loop diagrams enter only at fifth order, while the fourth-order Lagrangian is still in the process of being worked out [11]. There are however some processes where the leading contribution is at two loop; an example is the imaginary part of the nucleon electromagnetic form factors, which involves the anomalous $\gamma \to 3\pi$ vertex, with all the pions coupling to the nucleon. This process was considered by Bernard *et al.* in ref. [12]. However since the imaginary part comes from the kinematical regime where the pions are on-shell, this is not a two-loop calculation in the sense of having two internal momenta to integrate over. The first calculation of this type was the fifth-order piece of the chiral expansion of the nucleon mass [13], in which the only non-vanishing contribution came from the expansion of the relativistic one-loop graph in powers of $1/M_N$. The current paper presents another two-loop calculation, namely the leading (seventh-order) contribution of the anomalous Wess-Zumino-Witten (WZW) Lagrangian to forward spin-dependent Compton scattering. By the LET of Low, Gell-Mann and Goldberger [1] discussed above, this should of course vanish. We find that it does so, quite non-trivially, and the result enhances our confidence in the consistency of HBCPT, as well as showing the compatibility of the WZW Lagrangian and HBCPT, and testing the structure of the former in a novel way.

For our calculation we start from the anomalous Lagrangian as given by Witten [14], but for consistency with ref. [15] we use the opposite convention for the sign of e, namely $D^\mu = \partial^\mu - eA^\mu[Q, .]$. Thus we obtain the following Feynman rules; for $\gamma \to 3\pi$,

$$\frac{eN_c}{12\pi^2 f_\pi^2}\epsilon_{\alpha\beta\gamma\delta}\epsilon^{abc}\epsilon^\alpha k_1^\beta k_2^\gamma k_3^\delta, \tag{3}$$

and, in the sigma representation used in ref. [15], for $2\gamma \to 3\pi$,

$$i\frac{e^2N_c}{24\pi^2 f_\pi^2}\epsilon_{\alpha\beta\gamma\delta}(q_1-q_2)^\alpha\epsilon_1^\beta\epsilon_2^\gamma\Big(\delta^{ab}\delta^{c3}(k_1+k_2-k_3)^\delta+\delta^{ac}\delta^{b3}(k_1+k_3-k_2)^\delta +\delta^{bc}\delta^{a3}(k_2+k_3-k_1)^\delta\Big), \tag{4}$$

where the q_i and ϵ_i are photon momenta and polarisation vectors, and the k_i are pion momenta; all momenta are outward going. (In the exponential representation more frequently used for purely pionic processes, the latter expression gains an overall factor of $\frac{2}{3}$, and the pion momenta enter in the combination $(k_1 + k_2 - 2k_3)$ etc.) Equation (3) agrees with the expression given by Bernard *et al.* in ref. [12].

Classes of diagrams which enter in the calculation of the anomalous forward-scattering amplitude are shown in fig. 1. There are of course many other two-loop diagrams and other contributions at this order, but we are interested only in those with an anomalous vertex (denoted by a solid dot), which can be distinguished from all others by giving an amplitude proportional to N_c. All other vertices are taken from $\mathcal{L}^{(1)}_{\pi N}$, as listed in ref. [15], to give

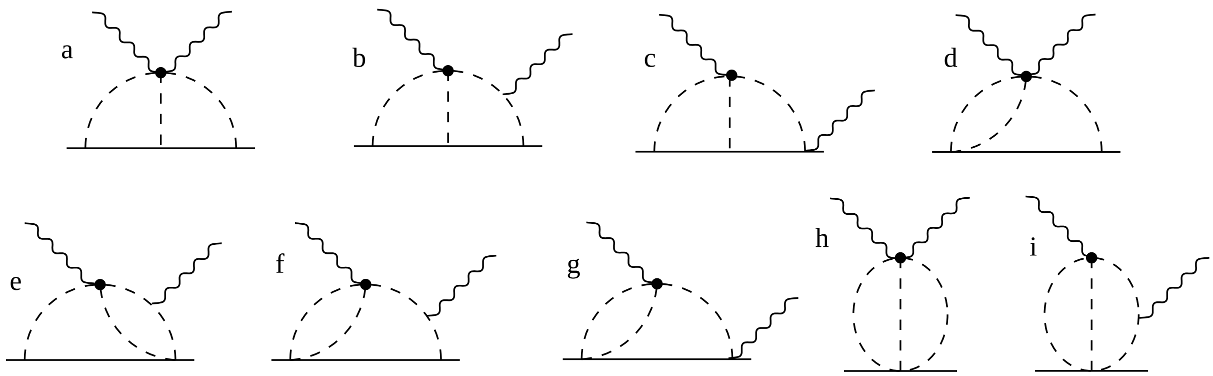

Fig. 1: Diagrams with anomalous vertices which contribute to Compton scattering in the $\epsilon \cdot v = 0$ gauge.

the lowest order contributions. By working in a transverse gauge for which $\epsilon \cdot v = 0$, we can immediately drop diagrams in which one of the photons couples directly to the nucleon; vertices in which one photon and two or three pions couple to a nucleon at a point also vanish; none of these diagrams is shown. Figs. 1a-c are all proportional to g_A^3, while figs. 1d-i are proportional to g_A; the two groups may therefore be considered separately. Diagrams with only one pion exchanged between the anomalous vertex and the nucleon do not contribute to forward scattering.

The spin-dependent forward scattering amplitude of Eq. (1) becomes, in the conventional covariant notation of HBCPT,

$$\epsilon_2^\mu \Theta_{\mu\nu} \epsilon_1^\nu = 2ie^2 v \cdot q W^{(1)}(v \cdot q) \epsilon_{\alpha\beta\gamma\delta} v^\alpha \epsilon_2^\beta \epsilon_1^\gamma S^\delta + \ldots \tag{5}$$

The low energy theorem states that $W^{(1)}(0) = -e^2\kappa^2/2M_N^2$, and this is satisfied at third order in HBCPT with κ replaced by its bare value. The only contributions to $W^{(1)}(0)$ at higher order that would not violate the LET therefore are those that renormalise κ. To test this we need only calculate the derivative of the amplitude with respect to $v \cdot q$ at $p = q = 0$ (where p is the nucleon residual momentum), and pick out the spin-dependent part.

We will consider the contribution from fig. 1a-c first. All the integrals which finally have to be evaluated have the general form

$$\int \frac{d^dk\, d^dl}{(2\pi)^d} \frac{l^\mu l^\nu \ldots k^\alpha k^\beta \ldots}{(v \cdot l - i\epsilon)(v \cdot k - i\epsilon)(l^2 - m^2 + i\epsilon)^a((l-k)^2 - m^2 + i\epsilon)^b(k^2 - m^2 + i\epsilon)^c} \tag{6}$$

where a, b and c are 1 or 2, with $a+b+c \leq 4$, and there are up to six powers of k or l in the numerator. These terms in the numerator come from terms such as $\epsilon \cdot l$, $S \cdot l$, or $\epsilon_{\alpha\beta\gamma\delta} v^\alpha l^\beta \ldots$, and so are all perpendicular to v. In the absence of external momenta, the only Lorentz structures possible are v^μ and $g^{\mu\nu}$; here we need, of the many structures possible with up to six indices, only the ones constructed without v^μ. These may be isolated by contracting with the projector $v_\mu v_\nu - g_{\mu\nu}$. The reduction to pure denominator form involves tedious but relatively straightforward algebra. The integrals which arise are as follows.

Two one-loop integrals enter,

$$J = i\int \frac{d^dk}{(2\pi)^{d/2}}\frac{1}{v\cdot k\,(m^2-k^2)} \qquad \text{and} \qquad \Delta = i\int \frac{d^dk}{(2\pi)^{d/2}}\frac{1}{(m^2-k^2)}. \tag{7}$$

The two-loop scalar integrals which appear fall into three classes. One has denominators like that of eq. 6, with the powers a, b and c being 0, 1 or 2, with $a+b+c \leq 4$; these are denoted $R(a,b,c)$. (Of course $b \neq 0$, or the integral is reducible.) The identity

$$\frac{1}{v\cdot l\;\; v\cdot k} = \frac{1}{v\cdot l\;\; v\cdot(k-l)} - \frac{1}{v\cdot(k-l)\;\; v\cdot k} \tag{8}$$

can be used to show that $R(0,1,0)=0$ and $R(1,1,0)=-J^2/2$.

The second class comprises simply the two purely mesonic integrals

$$S_1 = \int \frac{d^dk\,d^dl}{(2\pi)^d(l^2-m^2)(k^2-m^2)((l-k)^2-m^2)}, \tag{9}$$

and S_2 which is like S_1 but with one of the denominators—it doesn't matter which—squared. The two integrals are not independent; $S_2 = (1/3)(dS_1/dm^2) = (d-3)S_1/3$.

The last integral arises in the evaluation of fig. 1b, and is the only one in which the numerator cannot be eliminated by algebraic means:

$$\begin{aligned} T &= \int \frac{d^dk\,d^dl\,v\cdot l}{(2\pi)^d v\cdot k\;(l^2-m^2)^2((l-k)^2-m^2)} \\ &= -\frac{1}{2}\int \frac{d^dk\,d^dl}{(2\pi)^d(v\cdot k+v\cdot l)^2(l^2-m^2)(k^2-m^2)} \end{aligned} \tag{10}$$

The second line follows from the use of integration by parts, after shifting $k \to k+l$ and writing

$$\frac{v\cdot l}{(l^2-m^2)^2} = -\frac{v_\mu}{2}\frac{\partial}{\partial l_\mu}\left(\frac{1}{l^2-m^2}\right). \tag{11}$$

By further lengthy manipulation, or by direct evaluation using Feynman parameters, it is possible to show that this integral reduces to $-(d-2)^2\Delta^2/(4m^2(d-3))$, but we do not need this result.

Such integration-by-parts identities are crucial in the final step, which is to reduce the number of independent integrals which enter. Diagram 1b is the hardest to evaluate; of the two irreducible loop integrals, 1a and 1c involve only $R(1,1,1)$, and S_1, but 1b brings in $R(2,1,0)$, $R(2,1,1)$ and T. A series of integration-by-parts identities can be used to show the following relation:

$$6m^4R(2,1,1) + 2m^2R(2,1,0) + 4T = 2(d-4)m^2R(1,1,1) + 2(d-2)S_1 - (d-3)J^2. \tag{12}$$

This is sufficient to eliminate these extra integrals, after which it can be confirmed that the total vanishes. (It is also possible to prove that $m^2R(2,1,0) = T-(d-3)J^2/4$, and hence express $R(2,1,0)$ purely in terms of one-loop integrals, but again we do not need this.)

This leaves diagrams 1d-i. The integrals entering in the evaluation of these diagrams have at most one heavy-baryon propagator, and after reduction to scalar form only the purely mesonic integrals S_1 and Δ appear. Again, the total vanishes.

Thus even in this complicated process, with anomalous vertices and two-loop diagrams, the low energy theorem of Low, Gell-Mann and Goldberger is satisfied. Of course, since it is based only on Lorentz and gauge invariance, it would have been chiral perturbation theory and not the LET that would have suffered if it had been otherwise, but as this is the first test of the consistency of HBCPT to two loop order, the agreement is gratifying.

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APPENDIX

After reduction to scalar form and using identities such as the relation between S_1 and S_2, but before the application of any of the integration-by-part identities, the relevant contributions of diagrams 1a-i to the spin-dependent forward scattering amplitude have the following form:

$$W^{(1)}(0) = \frac{gN_c}{24\pi^2 f^3} M^n \tag{13}$$

where M^n is a combination of scalar integrals, listed below; $M^g = 0$.

$$\begin{aligned}
M^a &= \frac{g^2}{f^2(d-1)}\Big(\frac{1}{d}(d-2)(m^2 S_1 - \Delta^2) - 2m^2 J^2 + 2m^4 R(1,1,1)\Big) \\
M^b &= \frac{-g^2 m^2}{(d-1)(d-2)f^2}\Big(4m^2 R(1,1,1) - (d-1)J^2 + 6m^4 R(2,1,1) + 2m^2 R(2,1,0) + 4T\Big) \\
&\quad + \frac{g^2}{d(d-1)f^2}\Big((d+2)m^2 S_1 + (d-2)\Delta^2\Big) - M^c \\
M^c &= \frac{2m^2 g^2}{(d-1)(d-2)f^2}\Big(2R(1,1,0) + 3m^2 R(1,1,1) - J^2\Big) \\
M^d &= \frac{1}{d(d-1)}\Big((2d-3)\Delta^2 + 3m^2 S_1\Big) \\
M^{e+f} &= -M^d \\
M^h &= \frac{1}{d}\Big(m^2 S_1 - \Delta^2\Big) \\
M^i &= -M^h
\end{aligned} \tag{14}$$

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